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# SOLAR MODULATION OF COSMIC RAYS AND ITS RELATIONSHIP TO PROTON AND HELIUM FLUXES, INTERSTELLAR TRAVEL, AND INTERSTELLAR SECONDARY PRODUCTION

BY
N. DURGAPRASAD
C. E. FICHTEL
D. E. GUSS

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

# Solar Modulation of Cosmic Rays
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N. Durgaprasad[1], C. E. Fichtel, and D. E. Guss

NASA, Goddard Space Flight Center

Greenbelt, Maryland

## Abstract

The general problem of solar modulation of the galactic cosmic radiation is examined with particular emphasis on the solar wind model and a comparison to the experimental cosmic ray data obtained during the period from 1959 through 1965 including new results by the authors. It is first shown that the experimental data available at present allows only limited conclusions to be drawn without making any restrictions at all on the spectra outside the solar system. The analysis then shows that the assumption of similar energy/nucleon spectra at the source for protons and helium nuclei leads to unmodulated spectra which have very high intensities at low energies and a source spectral shape for helium nuclei which is inconsistent

[1]On leave of absence from the Tata Institute of Fundamental Research, Bombay, India.

with the results deduced from an analysis of helium and heavier nuclei. The assumption of similar rigidity spectra at the source leads to a more reasonable possible source spectral shape. The effect of the production of low energy protons in interstellar space is included in the analysis. Finally, the degree of modulation and a proton source spectrum are derived with the assumption that the helium spectrum deduced from an analysis of helium and heavier nuclei is correct. This final analysis leads to proton and helium nuclei source spectra which are markedly different from each other.

## I. Introduction

The intensity and energy spectra of the low energy galactic cosmic radiation is known to vary strongly with time, being roughly inversely correlated with solar activity. Several theories have been proposed to explain these variations, and it is now known that most of them are unsuccessful. In this paper, the results of work undertaken by the authors will be combined with other data which have been published to obtain a picture of the time variation of the cosmic radiation during the declining phase of solar activity (1959-1965). After a quick summary of the various theoretical approaches which have been taken to the problem of cosmic ray modulation, one particular model which seems particularly promising, the solar wind model (Parker, e. g., 1958a, 1965), will be explored and its predictions compared to the experimental data. To gain further insight into the problems of modulation and the source spectra, the effect of interstellar travel on the cosmic rays will be examined. The production of low energy protons in interstellar space and the implications of the relative spectral shape of the helium and heavier nuclei will be given particular attention. From this analysis tentative estimates will be made of the degree of solar modulation, the energy dependence of the modulation, and the shapes of the source spectra of the primary cosmic ray components.

## II. Experimental Data

In this section, we wish to summarize some of the experimental data which will be used in the subsequent discussion. Before proceeding to the general presentation of the data, experimental results obtained by the authors which have not previously been published will be briefly described so that they may be included with other differential flux measurements.

The experiments just mentioned were a part of a continuing program begun in 1961 to study the low energy galactic cosmic ray proton and helium nuclei spectra.

In this series nuclear emulsion detectors were flown on high altitude balloons from Fort Churchill, Manitoba at ceiling altitudes of between 2 and 3 gm/cm$^2$ of residual atmosphere for about 10 hours. The primary emulsion stacks were rotated into the exposure position at the balloon ceiling altitude in order to minimize the correction to the data for particles collected during balloon ascent. In addition drop stacks, supplementary emulsion stacks which were released from the balloon at ceiling when the main stack was rotated, were flown and used to provide the intensity of tracks collected on the ground and during the balloon ascent. The details of the balloon flights are shown in Table 1.

## A. Hydrogen Nuclei Data Analysis

The emulsions, except those from the 1962 flight, were scanned at two depths--one to provide data in the energy interval from about 60 to 80 MeV and the other to provide data in the range of energies from about 80 MeV to 250 MeV. The upper limit of the energy was set so that particles would change grain density sufficiently in the stack to permit a determination of their direction of motion. In general two corrections were applied to the data: (a) a correction for particles collected on the ground and during balloon ascent, and (b) a correction for secondary particles produced in the residual atmosphere above the balloon ceiling altitude. Because of an unusual ascent period an additional correction was required for the 1961 flight as considered in detail in paper 1 (Fichtel et. al., 1964a).

The background-ascent correction was made in each case by subtracting the suitably normalized drop stack spectrum from the spectrum obtained from the primary stack.

The correction for production of secondary particles in the atmosphere was obtained by analyzing the tracks which lay in the acceptance solid angle at the scan line and resulted from interactions in the emulsion between the scan line and a line 8.5 mm above the scan line. The relation between secondary particles produced in emulsion and those produced in air has been considered in detail in paper 1. In applying this correction to the data, the spectral shape of the secondary spectrum was obtained by combining the secondary spectra from the various years. The spectrum of emulsion secondaries which lay in our acceptance solid angle are shown in Figure 1. The intensity normalization for a particular year was obtained from the flux of secondary particles for that year except in 1962 where the secondary particles were not measured directly. For this latter case the intensity was determined by interpolating, using measured alpha particle intensities as a normalization parameter. The correction for secondary particle production in the obscured edge of the emulsion and in the atmosphere was made in two steps. The contribution from the emulsion equivalent layer (that is, the layer consisting of the obscured edge of the emulsion and an additional amount of overlying atmosphere which yields an ionization loss equivalent totaling 8.5 mm of emulsion) was calculated from the energy spectrum of emulsion produced secondaries as observed at the scan line. This typically corrected for the blackened emulsion edge plus about 2.2 $gm/cm^2$ of air, the exact amount depending upon the amount of edge blackening in a particular stack. The contribution of secondary production in the remaining atmosphere, overlying the emulsion equivalent layer, was made by breaking it into several intervals. In each interval the secondary production spectrum was assumed to be that of the emulsion secondaries at their interactions as shown in Figure 1, and the intensity was normalized using the observed intensity of emulsion secondaries with sufficient energy to penetrate the emulsion equivalent layer and the parameters, described in paper 1, relating secondary particle production in emulsion to that in air.

Throughout the data there was no attempt to determine the isotopic composition of the singly charged particles. The deuterons and tritons which were identified by ionization versus range measurements were put into the energy interval corresponding to their energy per nucleon, which is consistent with the energy intervals into which unidentified deuterons and tritons would be put, since their energy would be determined by ionization rather than range. For particles with range in emulsion less than 2 cm an accurate isotope determination could not be made, and these particles were assigned an energy on the basis of range, assuming them to be protons.

B. Helium Nuclei Data Analysis

The helium nuclei data were handled in a manner described in detail in paper 1. The only exception to this is that in the 1963 and 1964 flights the emulsion stacks were oriented at $90^{\circ}$ to the vertical during the ascent period so that the ascent correction was very much simplified. In these latter two years, it was assumed that no alpha particles entered into the acceptance solid angle during ascent.

The final spectra for hydrogen and helium nuclei for the years 1961 through 1964 are tabulated in Table II. Small changes in the proton fluxes from those published previously, (Fichtel et al., 1964a, 1964b), result from the new correction for secondary protons produced in the atmosphere above the emulsion stack incorporating all of the data on secondary protons produced in emulsion. The quoted standard deviations for the proton data include the statistical errors in the atmospheric secondary correction, the ascent correction, and the primary data. These data, combined with those of other investigations for the period between 1959 and 1965 are presented in Figures 2 and 3.

## III. Cosmic Ray Modulation Theories

The problem of the modulation of cosmic rays is one that has been studied extensively for a considerable period of time. There are several papers which review and discuss the various models in some detail, some of the more recent ones being Webber (1962), Dorman (1963), Webber and McDonald (1964) and Fichtel et al., (1964a). With the recent detailed observations in space of magnetic fields (Ness et al., 1964 and Ness and Wilcox, 1965, Wilcox and Ness, 1965, Davis et al., 1966, Coleman et al., 1966a, b) and plasmas (Neugebauer and Snyder, 1962, Snyder et al., 1963, and Bridge et al., 1965), the solar wind theory of the modulation of cosmic rays seems now to be on firm experimental ground, whereas all of the other modulation models have serious difficulties which are discussed in the reviews mentioned above. The specific difficulties with two of the more frequently discussed alternate models will be presented briefly below,

TABLE I. Balloon Flight Data

| Date of Flight | Mt. Washington Neutron Monitor Rate | Mean Altitude $g/cm^2$ | Time of Exposure (Sec x $10^4$) |
|---|---|---|---|
| 7 July 1961 | 2148 | 2.43 | 3.534 |
| 28 July 1962 | 2235 | 4.03 | 3.444 |
| 15 June 1963 | 2320 | 3.12 | 4.032 |
| 21 June 1964 | 2391 | 2.64 | 3.540 |

TABLE II. **Differential Fluxes of Singly Charged Particles and Helium Nuclei from 1961 through 1964.**

Singly Charged Particles

| 7 July 1961 | | 28 July 1962 | | 15 June 1963 | | 21 June 1964 | |
|---|---|---|---|---|---|---|---|
| Mev | $p/m^2sr.sec$ | Mev | $p/m^2sr.sec$ | Mev | $p/m^2sr.sec$ | Mev | $p/m^2sr.sec$ |
| 68.1 | $0.019\pm0.41$ | - | - | 74.5 | $0.85\pm0.55$ | 71.2 | $0.12\pm0.86$ |
| 120 | $0.25\pm0.13$ | 120 | $0.26\pm0.18$ | 120 | $0.87\pm0.22$ | 120 | $0.94\pm0.35$ |
| 205 | $0.56\pm0.12$ | 205 | $0.47\pm0.13$ | 185 | $0.82\pm0.25$ | 205 | $1.21\pm0.29$ |

Helium Nuclei

| 7 July 1961 | | 28 July 1962 | | 15 June 1963 | | 21 June 1964 | |
|---|---|---|---|---|---|---|---|
| Mev/Nuc | $p/m^2sr.sec$ Mev/nuc | Mev/Nuc | $p/m^2sr.sec$ Mev/nuc | Mev/Nuc | $p/m^2sr.sec$ Mev/nuc | Mev/Nuc | $p/m^2sr.sec$ Mev/nuc |
| 83 | $0.062\pm0.024$ | 113 | $0.156\pm.031$ | 108 | $0.232\pm0.040$ | 109 | $0.298\pm0.062$ |
| 150 | $0.073\pm0.012$ | 200 | $0.213\pm0.032$ | 200 | $0.257\pm0.043$ | 175 | $0.340\pm0.085$ |
| 250 | $0.100\pm0.013$ | 300 | $0.213\pm0.032$ | 300 | $0.215\pm0.030$ | 225 | $0.468\pm0.100$ |
| 350 | $0.140\pm0.015$ | 400 | $0.158\pm0.027$ | 400 | $0.205\pm0.037$ | 300 | $0.361\pm0.062$ |
| 450 | $0.150\pm0.017$ | 500 | $0.141\pm0.025$ | 500 | $0.170\pm0.030$ | 400 | $0.223\pm0.049$ |
| 550 | $0.075\pm0.023$ | - | - | - | - | 500 | $0.361\pm0.062$ |

but the main portion of this section will be devoted to a review of the solar wind modulation theory in preparation for the comparison to the experimental data which will be developed in the next two sections. The solar wind modulation theory was first proposed by Parker (1958a), based on the theoretical prediction of the general nature of the interplanetary magnetic field (Parker 1958 b and c), and has subsequently been developed in detail by Parker (1963, 1965), Axford (1965), and Fibish and Abraham (1965).

## A. Alternate Theories

The two modulation mechanisms mentioned above which are significantly different from the solar wind modulation are electric deceleration and a disturbed solar dipole. The former was first proposed by Nagashima (1953) using a geocentric model, and later used by Ehmert (1960) in a high helio-centric model. This theory has subsequently been explored and compared to experimental data by McDonald and Webber (1959), Fichtel (1961), Freier and Waddington (1965a) and others. There are a number of objections to the electric deceleration model in addition to its failures to give entirely satisfactory agreement with experimental data. These included the following: the electrical conductivity in interplanetary space will almost certainly not support the necessary potential, electron data disagrees with the existence of a large potential (For a recent survey, see Abraham, Brunstein, and Cline 1966), and the detailed characteristics of the heavy nuclei argue against a large deceleration (Fichtel and Reames 1966).

The modulation of the cosmic radiation by a solar dipole was first proposed by Janossy (1937) and extended by Elliott (1960). The principal arguments against this theory are the disagreement between the magnetic field configuration which is required by the theory and that which is observed, the incorrect prediction of the variation in intensity with distance from the sun, and the detailed assumptions regarding scattering into the sun which are thought to be unlikely.

### B. Solar Wind Modulation

In the solar wind picture the modulation is due to the diffusion and convection of particles in the presence of small scale irregularities in the magnetic field of the outward moving interplanetary plasma (Parker 1963, 1965). The random walk of a particle in these small scale irregularities can be determined once the pattern of irregularities and the large scale magnetic field has been decided upon. As mentioned previously the general picture of the interplanetary magnetic field at least in the vicinity of the earth is now clearly established as being the generally spiral pattern predicted by the solar wind theory with the superimposed irregularities expected from plasma instabilities,(Parker 1958b,1958c). Although the diffusion is not expected to be isotropic because of the large scale magnetic field, Parker (1965) has shown that, even for the two extreme cases of completely isotropic diffusion and diffusion constrained to lie along the large scale spiral magnetic line of force, it is not possible to distinguish the difference at the earth experimentally. Therefore, for the sake of simplicity, we shall begin with the isotropic case and then only later discuss the anisotropic case. In the solar wind modulation there is also a possible deceleration of the cosmic ray particles, and this effect will be discussed at the end of this section.

Ignoring the deceleration contribution, the differential intensity of particles in the vicinity of the earth for the isotropic case before it is affected by the earth's magnetic field, j(i,k), is given by the expression (Parker, 1963):

$$j(i,k) = g(i,k) \exp\left[ -\frac{3}{\beta}\int_{r_e}^{\infty} \frac{V(r)}{\lambda(R,r)}dr\right] \quad (1)$$

g(i,k) is the spectrum just outside the solar modulation region, i refers to the type of particle ( $\alpha$ for helium nuclei and p for protons), k refers to whether the spectrum is in energy per nucleon, E/N, or rigidity, R, V is the solar wind velocity, $\beta$ is the particle velocity in terms of the speed of light, r is the distance, and $\lambda$ is the mean free path.

In order to obtain a simpler function, we shall make the following two assumptions, which it is hoped will not introduce any gross error and will at least include most of the models which have been proposed. (1) $[1/\lambda(\vec{r}, R)]$ is a function only r and not $\vec{r}$, and (2) the variables r and R are separable, $1/\lambda$ may then be written as:

$$1/\lambda = G(r)\, H(R) \quad (2)$$

It should be noted that this equation includes the case where G(r) is a constant and G(r) is proportional to $1/r$ or $1/r^2$ discussed by Quenby (1966) as well as several other possibilities.

Substituting equation (2) into (1) yields

$$j(i,k) = g(i,k) \exp \left[-\frac{3}{\beta} \int_{r_e}^{\infty} V(r)\, G(r) dr\, H(R)\right] \qquad (3)$$

Since the integral is just a constant, we have

$$j(i,k) = g(i,k) \exp \left[-AH(R)/\beta\right] \qquad (4)$$

where $$A = 3\int_{r_e}^{\infty} V(r)\, G(r) dr \qquad (5)$$

The next problem is the determination of the rigidty dependence of $(1/\lambda)$, that is the function $H(R)$. Several different approaches will be explored here; then, finally a comparison will be made. For the spherically symmetric case, two limiting forms for $\lambda$ are $1/\lambda =$ constant, the strong scattering center approximation and $1/\lambda = (R_o/R)^2$, the weak scattering center approximation. In an early paper, Parker (1956) suggested the form

$$\lambda = \lambda_o \left[1 + \left(\frac{\pi\rho}{2L}\right)^2\right],$$

which for the purposes here, is equivalent to

$$H(R) = \left[1 + (R/R_o)^2\right]^{-1} \qquad (6)$$

$\rho$ is the radius of curvature, L the diameter of the scattering center, $R_o = 2BL/\pi$, and B is the magnetic field strength, since equation (6) reduces to the weak scattering approximation for large $\rho$, the strong one for small $\rho$, and also provides a reasonable transition between the two.

Parker (1963) later gave a more exact expression for the rigidity dependence of $\lambda$, which is equivalent to the following:

$$H(R) \sim \int \ell^2 N(\ell)\ f(\ell)\ d\ell \text{ where}$$

$$f(\ell) = 1, \text{ for } \ell > \rho \text{ and}$$

$$f(\ell) = [\ell/\rho]^2 \text{ for } \ell < \rho \qquad (7)$$

In this last set of equations, $N(\ell)$ is the size distribution of scattering centers.

In addition to looking at the recent data obtained during the declining phase of the solar cycle, it is interesting to explore the effect of various different choices of H(R) to see the importance of the size distribution. A series of possible $N(\ell)$ functions have been selected, and some of them along with the resulting H(R) functions are shown in Table III. The choice was limited to H(R) functions which had only one adjustable parameter $R_0$, the mutiple constant being absorbed in A. In general, not setting a lower cutoff to the $\ell$ distribution, e.g., $\ell o$, seemed to have only a very small effect on H(R) except for R values very close to the small rigidity corresponding to $\ell o$.

At this point, it is desirable to consider the fact that a large scale field does exist. Parker (1964) has shown that a charged particle moving along a field with small scale irregularities is most effectively scattered by irregularities which have a scale size comparable to the radius of gyration of a particle. Particles of appreciably smaller radius simply follow the field line and pass smoothly through the irregularity, and particles of appreciable higher rigidity are only slightly deflected. Thus, perhaps, if the distribution of scatterers

TABLE III. Table of Values of $H(R)$ for Various Assumed Scatter Center Size Distributions

| Symbol | $N(\ell)$ | $H(R)$** | |
|---|---|---|---|
| | | $R_o{}^* > R$ | $R_o < R$ |
| G | ——— | $[1 + (R/R_o)^2]^{-1}$ | |
| I | K, for $\ell < L$<br>O, for $\ell \geqslant L$ | $[1 - .4(R/R_o)^3]$ | $[.6(R_o/R)^2]$ |
| II | K, $\exp[-\ell/L]$ | $(R_o/R)^2 \{1-\exp(-R/R_o)[1+R/R_o+(10/24)(R/R_o)^2+(1/12)(R/R_o)^3]\}$ | |
| III | $K(1/\ell^2-1/L^2$ for $\ell < L$<br>O for $\ell > L$ | $\left[1-\left(\frac{R}{R_o}\right)+.2\left(\frac{R}{R_o}\right)^3\right]$ | $[.2(R_o/R)^2]$ |
| IV | $K(1/\ell^2-1/L)$ for $\ell < L$<br>O for $\ell > L$ | $\left[1-1.5\left(\frac{R}{R_o}\right)^2 + .8\left(\frac{R}{R_o}\right)^3\right]$ | $[.3(R_o/R)^2]$ |

* In the above, $R_o = BL$.

**Multiple constants are not included and are absorbed in A of equation (5).

is reasonably smooth, a fair estimate of the rigidity dependence of $\lambda$ for this case would be the dependence of the average distance between scattering centers on particle rigidity.

Before dicussing the results of analysis of the magnetic field data referred to earlier, it is perhaps worth reminding ourselves first that the present measurements are made in a spacecraft moving through space at a velocity very small compared to the solar wind and represent approximately the field in the plasma as it moves by one point in space. No true experimental picture of the field lines and their irregularities in the solar system exists. In particular, the variation of the irregularities with distance from the sun is not known. Secondly, all of the measurements have been made in the last several years and therefore reflect a relatively quiet solar period.

In order to obtain some idea of the regularity of the field, Parker (1965) created plots drawn by causing a pen to progress with constant speed across the paper (representing the plane of the ecliptic) in the direction of the interplanetary field at each instant of time. These plots indicated that there is apparently a continuous spectrum of irregularities of all scales decreasing in number with increasing size from the smallest observable ($\sim 1.2 \times 10^5$ km) up at least to a dimension corresponding to the radius of curvature of a particle of several tens of BV/c. The distribution of scattering sizes appeared to be well within the range of those given in Table III.

In order to make a more quantitative estimate of the distribution of the magnetic irregularities, Jokippi (1966) has used the power spectrum analysis made by Coleman (1966) on the Mariner II magnetic field data. He found that the scattering was primarily due to irregularities with scale sizes which are of the order of the radius of curvature of the particle in the magnetic field. Further from his analysis, Gloeckler and Jokippi (1966) have shown that the diffusion coefficient describing the motion of the cosmic rays during the period of low solar activity (1964 - 1965) is proportional to BR in the region of interest and that the diffusion coefficient along the field is very much greater than across it. Parker(1965) has already shown that in this case, i.e., diffusion primarily along a field line, the modulation equation is essentially identical in form to equation (4) except that now A is a more complicated function involving the velocity of rotation of the sun and the solar wind velocity. Hence, for the present, equation (4) can still be used with H proportional to R. We shall then come back later to the question of whether or not the quantitative values determined for A are reasonable.

So far the problem of energy loss has not been discussed. Parker (1965) has shown that the cosmic ray particles lose a significant amount of their energy to the expanding interplanetary fields. The amount of energy loss is a function of the initial energy and in the isotropic diffusion case on the parameter $r_o$ V/K, where K is the diffusion coefficient and except for sign is the quantity in square brackets in equation 1. The energy loss equation is solved only for large values of $r_0V/K$, and $r_0V/K=0$ and is interpolated in between It is a percentage of the total energy for a fixed $r_0V/K$ and depends somewhat on particle velocity, in addition to the velocity dependence in K, being a greater percentage for non-relativistic particles. Figure 6 of Parker (1965) shows the degree of the effect as a function of $r_0V/K$. To see $r_0V/K$ in the symbols being used here, we have $(r_0V/K) = 3r_0V/B\lambda = AH(R)/B$. A further discussion of deceleration and its importance will be given in the analysis of the experimental data (part VI).

It has been noted (e.g.,Webber and McDonald, 1964), that the neutron monitor data suggests that the modulation at high rigidities (5 to 50 BV/c) is not well represented by the mean free path between collisions being proportional to the square of the rigidity, as would be suggested by the high rigidity portion of some of the modulation functions discussed here, but actually has a less strong dependence on rigidity. The analysis of Manzano and Winckler(1965), for example, suggests that $\lambda$ would be proportional to $R^{1.5}$. There may, however, not be a discrepancy between the basic physical theory and experimental data, but only an unjustified mathematical approximation in this range. The $R^2$ dependence results from a random walk calculation wherein each deflection of a very large number is proportional to R.

If the number of significant deflections is small, as it may be as one approaches high rigidity, then, an $R^2$ dependence would not be expected, but a less strong dependence somewhere between R and $R^2$. Further the $R^2$ dependence occurs only in the case of isotropic scattering centers and an R dependence occurs in the case of an isotropic diffusion along a field line, as we have mentioned.

## IV. Comparison of Experimental Data to the Solar Wind Theory Without any Restrictions on the Spectra Outside of the Solar System

In this section we shall discuss the approach to modulation which is in principle the most attractive; that is, an analysis will be made of what can be learned without any assumptions being made about the spectrum outside the solar system. Examining equation (5), it can be seen that this approach implies the elimination of g(i,k) in some manner. If j(i,k) is measured at two different times, the expression for j(i,k) at one time can be divided by that at another. Performing this operation and taking the logarithm of the resulting expression yields:

$$\ln\{j(i,k,t_1)/j(i,k,t_2)\} = \frac{1}{\beta}\left\{A(t_2)\, H(R,R_o(t_2)) - A(t_1)\, H(R,R_o(t_1))\right\} \quad (8)$$

Since in the model proposed earlier only A and Ro may vary, a means of obtaining A and Ro as a function of time exists in principle since there are four values $A(t_1)$, $A(t_2)$, $Ro(t_1)$ and $Ro(t_2)$ for a given function $H(R,R_o)$, and the ratio $j(i,k,t_1)/j(i,k,t_2)$ can be evaluated over R. Further, the ratio can be examined for different species. In fact the limited data and large errors make this procedure very difficult, and full of pitfalls resulting from ambiguities consisting of several possibile interpretations depending on the assumptions related to $H(R,R_o,t)$.

Using this approach, Gloeckler and Jokippi (1966) have shown that there is a relatively strong rigidity dependence for the period from December 1963 to September 1965, in the energy interval from 20 to 90 MeV/nucleon both on the basis of the relative variations of the proton and helium fluxes and on the variation of the differential helium flux as a function of energy.

In their work they showed that $H \sim R^{-1}$ leads to satisfactory agreement with the data for both the changes from (December 1963 - May 1964) to (October - November 1964) and from (October - November 1964) to (June - September 1965). However, an analysis of the same data shows that an expression of the form of G on table I agrees with the data as well, as do several other representations for H. Specifically, using the work developed later as a guide to the numerical values, it was found that agreement could be obtained if $R_o$ changed from 0.85 BV/c for December 1963 - May 1964 to 0.6BV/c for October - November 1964 while A remained about 0.6.

Silberberg (1966) has used this approach to explore the data obtained during the period 1959 through 1964. He concludes that the data can be fitted by H equal to a constant below some rigidity and

and H proportional to $R^{-1}$ above, where the transition rigidity can vary with time. The data, however, also agree with other functions; for example, the expression $H = (1 + (R/R_o)^2)^{-1}$ will also satisfy the data (e.g., Fichtel et al, 1965).

In general, the limited data available do not really permit very full use of this approach during the years 1959 to 1963, and, even for the more accurate data obtained in 1964 and 1965, the interpretation is ambiguous.

V. Comparison of the Experimental Data with Solar Wind Theory with Restrictions on the Spectra Outside the Solar System

In this section a number of approaches will be considered. Firstly, it will be assumed that the source spectra of the protons, helium nuclei, and heavy nuclei are the same. It will be seen that this assumption leads to apparently irreconcilable contradictions. The treatment of interstellar travel and secondary production will also be presented and developed as necessary. Next, the assumption that the proton spectrum is the same will be abandoned, but the assumption that the spectra of multiply charged nuclei at the source are the same in shape will be retained. This is a not unreasonable assumption since the protons have a different charge to mass ratio from the remaining nuclei. The justifications for this assumption are treated in more detail by Fichtel and Reames (1966), though it is perhaps particularly worth noting that solar cosmic rays appear to have this property of similar spectra for all but the proton component (See for example Biswas and Fichtel, 1965)

## A. Similar Source Spectra

Before proceeding, the particular approach to be used will be discussed more exactly. It will be assumed that essentially nothing is known about the particular shape of the source spectrum outside of the solar system. Now, if only one type of particle, for example the proton, is considered, a wide class of modulation functions would be permitted since each could generate a potential source spectrum. The approach here will be to assume that the source spectrum of the protons and the helium nuclei are the same in energy/nucleon or in rigidity except for a normalization constant. The constraints are then fairly severe.

The problem will first be attacked considering only the effects of energy loss due to ionization in interstellar space in order to simplify the problem and make the significance of the result clear. The effects of fragmentation and low energy secondary proton production will then also be considered in the next subsection to show their effect.

Let us first look at the case of the identical energy/nucleon spectra at the source except for the normalization constant C. The energy/nucleon spectra just outside the solar system g(i,E/N) will also have the same form because the rate of energy loss in terms of of energy/nucleon is the same for protons and helium nuclei at a given energy/nucleon. Hence, from (4),

$$j(p,E/N) = g\ (E/N)\ \exp\left\{-(1/\beta)\ [AH(R_p,R_o)]\right\}$$

$$j\ (\alpha,E/N) = \frac{1}{C}\ g\ (E/N)\ \exp\left\{-(1/\beta)\ [AH(R_\alpha,R_o)]\right\}$$

$$\frac{j(p,E/N)}{j(\alpha,E/N)} = C\ \exp\left\{-\frac{A}{\beta}\ [H(R_p,R_o)-H(R_\alpha,R_o)]\right\} \qquad (9)$$

In 1963 there were several measurements of the proton and helium nuclei energy spectra, and we shall use these data for the discussion here, although data from other years would lead to conclusions, which are virtually identical. From these data, presented in Figures 2 and 3 it is possible to see if the theoretical picture will agree with the experimental data for any of the $\lambda$ functions chosen. It was found that a fit to the proton to helium ratio could be obtained as shown in Figure 4 for all but one of the forms of H(R) given in Table III, specifically G, II, III, and IV. Figure 5 shows the unmodulated spectra which result from this analysis and also the spectra after extrapolation through material back to the source. They are seen to be very steep spectra and would constitute a very large flux and energy density for cosmic rays; larger than magnetic field energy density in the galaxy, for example. Reasonable fits can be obtained for 1961, 1962, 1964, and solar maximum, using the ummodulated spectrum derived from the 1963 data and adjusting the parameters A and $R_o$ in the modulation functions. However, we shall not show these in a figure here due to the improbability of any of these source spectra being correct, since in addition to the objection just mentioned to this type of source spectra there is a strong argument against helium nuclei having this steep of a low energy spectrum based on analysis of the interstellar travel of helium and heavy nuclei (Fichtel and Reames, 1965).

Balasubrahmanyan et al. (1965) have shown that, if it is assumed that the proton and helium energy/nucleon spectra at the same at low energies only, then with a proton to helium ratio of five reasonable agreement can be obtained with the experimental data for a relatively flat source spectrum and a modulation function wherein H of equation (2) is a constant. This analysis, of course, implies different source spectra in order to account for the fifteen to one proton to helium nuclei ratio at high energies. The case of possibly different source spectra is analyzed in section V. C.

The second similar spectra assumption to be explored is that the protons and helium nuclei have similar rigidity spectra at the source. Here, the problem is not quite so straightforward because the particles lose different amounts of rigidity while traversing the same amount of interstellar material. To obtain an expression similar to equation (9), let us proceed as follows:

The source spectrum $j_S(i,R_i)$ is related to the local but unmodulated spectrum $j_L(i,R_{Li}) = j_S(i,R_{Si}) \dfrac{\Delta R_{Si}}{\Delta R_{Li}}$ (10)

Let $j_E(i,R_{Ei})$ represent the modulated spectrum observed at the earth, but unaffected by any earth field effects. $R_{Ei} = R_{Li}$ since there is no change in rigidity during this phase. Then,

$$j_E(i,R_{Ei}) = j_S(i,R_{Si}) \quad \frac{\Delta R_{Si}}{\Delta R_{Ei}} * \exp\left[ - \frac{1}{\beta_{Ei}} AH(R_o,R_{Ei})\right] \quad (11)$$

The assumption that the source rigidity spectra are identical in shape gives:

$$Kj_S(\alpha, R_{S\alpha}) = j_S(p,R_{Sp}) \quad (12)$$

Choose $R_{sp} = R_{s\alpha}$ and hence $\Delta R_{sp} = \Delta R_{s\alpha}$, and divide equation (11) for $i = p$ by equation (11) for $i = \alpha$ to obtain:

$$\frac{j_E(p,R_{Ep})}{j_E(\alpha,R_{Ep})} = K \frac{\Delta R_{E\alpha}}{\Delta R_{Ep}} * \exp\left[ - \frac{1}{\beta_{Ep}} AH(R_o,R_{Ep}) + \frac{1}{\beta_{E\alpha}} A(R_o,R_{E\alpha})\right] \quad (13)$$

Assuming the value of interstellar matter traversed taken from other work, it is then possible to determine exactly $R_{Ep}$, $R_{E\alpha}$, $\Delta R_{E\alpha}$, $\Delta R_{Ep}$, $\beta_{Ep}$, and $\beta_{E\alpha}$ for a given $R_{sp} (=R_{s\alpha})$ and then proceed as before.

Figure 6 shows the agreement between the proton to helium nuclei flux ratios measured in 1963 and the values predicted by equation (13) for the H(R) functions which give agreement with the data and an interstellar path length of 2.5 g/cm$^2$. The results are plotted as a function of the proton rigidity at the earth. Figure 7 then shows the source rigidity spectra for some of these values of H(R), and also shows the agreement between the theoretically predicted modulated spectra for $H_G$ and C = 6.5 as an example, and data obtained in other years. It is seen that good agreement as obtained, except for the increase in the hélium nuclei at very low rigidity in 1965, and a generally poor fit at the lowest rigidity point.

B. Similar Source Spectra and Interstellar Interactions

In the previous subsection, the effect on energy loss by ionization was considered, but the effect of fragmentation and secondary production was ignored. The effect of fragmentation has been considered in detail by Fichtel and Reames (1966) for particles with charges of two or more. The equations can be used here directly, and the results extended to singly charged particles. The final equation they obtain which will be of use to us here is:

$$\frac{d}{dx}\left[\exp\left(\frac{x}{\Lambda_i(E)}\right) j_i(E,x)\ w_i(E)\right] = \exp\left(\frac{x}{\Lambda_i(E)}\right) w_i(E)\ S_i(E,x) \tag{14}$$

where x is the position along the particle path, $w_i = (dE/dx)_i$, i refers to the particular nuclear species, $j_i$ is the differential directional intensity per unit energy/nucleon, $\Lambda_i$ is the loss mean free path, and $S_i$ is the source term. $\Lambda_i$ is related to the interaction mean free path, $\lambda_i$, by the equation

$$1/\Lambda_i = (1-P_{ii})/\lambda_i \tag{15}$$

where $P_{ii}$ is the average number of particles of type i formed in the interaction of a type i nucleus. $S_i$ is given by the relation

$$S_i(E,x) = \Sigma_{k>i}\ j_k(E,x)/\Lambda_{ki}(E) \tag{16}$$

where $\Lambda_{ki}$ is the mean free path for production of i-type particles from k-type particles. In the above fragmentation equations, it is always assumed that the parent and daughter nuclei have the same energy/nucleon.

For the case of cosmic ray protons there is another important class of interactions, those which lead to low energy protons whose energies are typically small compared to the primary. The production of secondary protons has been considered in detail by Feit and Milford (1965). Both the elastic and inelastic cross sections have been computed using the method described by these authors.

To include the production of low energy secondary particles, equation 14 may be used directly, but equation 16 must be modified to include the effect. Hence, the expression for S which includes both the effects is:

$$S_i(E,x) = \Sigma_{k>i}\, j_k(E,x)/\Lambda_{ki}(E) + \Sigma_{k\geq i}\int j_k(E',X)/\theta_{ki}(E',E)\, dE' \tag{17}$$

where $1/\theta(E',E)$ is the interaction mean free path for the production of i-type nuclei of energy E in dE from k-type nuclei of energy E' in dE'. The limited data available makes it reasonable to replace the integral by a sum. Further, because of the lack of information on cross sections and the belief that other contributions are small, only the production of low energy protons by helium nuclei and protons was considered. The values used are given in table IV. Also given in table IV are the fragmentation parameters which are those determined by Badhwar and Daniel (1963) for the calculation of interest here.

The particles have been placed in three groups, protons, helium nuclei, and ($Z \geq 3$) nuclei for the determination of the final proton

TABLE IV.

| $\Delta E$ (MeV) \ $\Delta T$ (MeV) | 10 - 90 | 90 - 250 | 250 - 500 |
|---|---|---|---|
| 250 - 1000 | 0.00012 | 0.000078 | 0.000048 |
| > 1000 | 0.00019 | 0.000054 | 0.000032 |

Average number of secondary protons produced within a one MeV interval in the kinetic energy range $\Delta T$ for a typical primary proton in the range $\Delta E$ per $g/cm^2$ of interstellar travel.

spectra. The calculations for helium spectra were performed in the manner described by Fichtel and Reames (1966) using five groups, helium, $(3 \leq Z \leq 5)$, $(6 \leq Z \leq 9)$, $(10 \leq Z \leq 19)$, and $(Z \geq 20)$ nuclei, and the fragmentation parameters used in that paper.

The results of the above calculation for protons and helium nuclei are shown in figures 8a and 8b for three different source spectral shapes both with and without the low energy secondary correction. The following conclusions can be drawn:

1. The fragmentation correction alone in interstellar space has a negligible effect on the ratio of the proton to helium nuclei as a function of energy after passage through $2.8 g/cm^2$ of material.

2. The low energy secondary proton correction has the effect of enhancing the proton to helium nuclei ratio at low energies, but this effect is significant only for relatively flat source spectra such as that shown in figure 8a. (Presumably, if the low energy helium nuclei secondary production were included this ratio would not be enhanced so greatly; however, although this effect cannot be calculated accurately it appears that it should be such that the low energy helium nuclei are enhanced by a lesser percentage than the protons from the nature of the interactions and the fact that interstellar nuclei consist primarily of protons.)

We can see immediately now that the result of including the effect of interactions in interstellar space in the consideration of the case of similar source spectra is to require even steeper source spectra than those in VA. This last statement is true because now C in equation (9) becomes a function of energy and is always equal to or greater than the value used formerly, having a tendency to increase towards lower energies. Therefore, the observed spectra must have been modulated more strongly. However, notice also that since the source spectra derived in VA were already very steep the calculations above show that the effect is really quite small. Hence, the primary conclusion of this subsection is that, including the low energy interstellar secondary effect, has essentially no effect on the result of subsection VA, namely that if the source spectra are similar they are very steep of the form of figures 5 or 7.

### C. Restricted Helium Spectrum

Another approach to the problem of trying to find a solution to the modulation problem is to abandon the restriction that the protons and helium nuclei must have similar spectra at the source, and rather to demand only that nuclei of the same charge to mass ratio have the same energy/nucleon spectra initially. Thus, for example, helium, carbon, oxygen, and neon would all have the same energy/nucleon or rigidity spectra, but the spectra of protons and helium nuclei might be quite different. There is good reason for making this assumption on the basis of accelerating mechanisms for reasons outlined by Fichtel and Reames (1966). Using this approach, they found that the relative abundance as a function of energy observed at the earth demand that the source spectrum of the helium and heavier nuclei must be quite flat at the source if they are to be the same. This conclusion arises principally from the amount of Li, Be, B, and $He^3$ determining the amount of matter traversed by the nuclei, and the near equality of the relative abundance at low energies and high energies. The maximum cosmic ray spectrum observed in 1965 sets a lower limit for the spectral flux at low energies; so the spectrum at the source for helium and heavier nuclei is defined within reasonably narrow limits. The source spectral form

$$\frac{dJ}{dE} = A_k/W_N^{\alpha} \qquad (18)$$

is in agreement with these constraints. Here, $W_N$ is the total energy/nucleon, $A_k$ is a constant for a given nuclear species, and $\alpha$ is a constant in the range of 2.5 to 2.7. Starting with this spectrum and extrapolating it through 2.8 g/cm$^2$ of interstellar space, in the manner described by Fichtel and Reames (1966), a spectrum is obtained representing the spectrum outside of the solar system in the same manner described previously. The modulating function can then be applied to this spectrum. Using form G for H(R) from table III with the assumptions implied in equation 4, it is possible to obtain agreement between the curves calculated in this manner and the experimental helium nuclei data. These are shown in figures 9a and 9b with the value of A and $R_o$ of equation 9 being indicated. It is also possible to obtain agreement with the experimental data for forms I, II, III, and IV for H(R) as given in table III. Hence, at least within the limits of variation implied by these forms the result seems to be insensitive to the type of distribution of scattering sizes chosen.

Having now determined the modulation function, it is possible to apply it together with the existing proton data to obtain an unmodulated proton spectrum outside of the solar system. This procedure was followed using the proton data from 1963, shown in figure 2, and the modulation parameters determined from the helium data. The spectrum determined in this manner was then modulated using the modulation parameters for the years 1965, 1963, 1961, and 1959 determined from the helium nuclei data to determine if the approach is consistent. Figure 10, except for the 1959 data, shows that it is. A somewhat better fit to the proton data in 1959 can be obtained using a higher value of A and a lower value of $R_o$, but the fit to the helium data is less satisfactory. At this time, this difficulty is not thought to be a serious objection to this particular approach due to the large uncertainty in the proton data points resulting from the relatively greater amount of residual atmosphere above the detector on these balloons flights compared to later flights and the strong possibility of only partial transmission of the low energy protons through the earth's magnetic field at the location of this flight (International Falls), compared to the later flights at Fort Churchill. This last point is felt to be particularly significant.

If the assumptions of this approach are correct, then, the results would indicate that, if the higher energy data (5 to 50 BeV/nucleon) are correct, the proton and helium nuclei have significantly different spectra.

This derived proton spectrum can now be extrapolated back to the source using the techniques discussed earlier. Including both the effects of fragmentation and low energy secondary production, the spectrum shown in figure 11 is obtained along with the assumed spectrum for helium nuclei. This figure shows that the proton to helium ratio at the source would be about 5 at a few hundred MeV/nucleon compared to about 15 to 20 at very high energies/nucleon.

## D. Value of the Modulation Coefficient

It is now possible to consider whether or not the experimental value obtained for A of equations (4) and (5) is reasonable in terms of what is known about the parameters involved in A. The exact function for A depends on the particular model chosen, so it will be necessary to make some specific choices. Beginning with the simplest case, if we assume G(r) of equation (2) is a constant, $G_o$, until some radius $r_o$ and zero thereafter, equation (5) gives

$$A = 3V\, G_o(r_o - r_e) \qquad (19)$$

As was seen above, A is approximately equal to 0.5 to 1 c, where c is the velocity of light, and V is approximately 400 Km/sec. Hence,

$$G_o(r_o - r_e) = (2.5 \text{ to } 5) \times 10^2$$

Since H(R) is of the order of unity for particles in the range of 1 BV/c equation (2) shows that $1/G_o$ gives the order of magnitude of

$\lambda$. Hence, if $\lambda$ for a 1 BV/c particle ($\rho = 0.7 \times 10^6$ KM in a $5\gamma$ field) is of the order of several million kilometers, $r_o$ is of the order of 5 to 20 earth radii, which is in the range normally suggested (e.g. Parker, 1963). As indicated earlier, Parker (1965) has shown that the results for diffusion essentially along a spiral field line lead to an equation similar to the isotropic case (See Parker 1965, equation 31). For values of $\lambda$ of the order discussed here, the equation for the strongly anisotropic diffusion analysis would indicate a somewhat smaller boundary to the scattering region. Naturally if the distance between scatters is assumed to decrease with increasing r, the calculated boundary will be further from the sun.

## VI. Comparison of the Experimental Data to the Solar Wind Theory Including Deceleration

As indicated in section III the problem of deceleration has not been solved exactly theoretically and so only an estimate of its effect exists. As an example, the 1963 spectra will be examined. The earlier estimates of A indicated that it was about 0.6; so, using the work of Parker (1963), an estimate of the deceleration effect can be made. The deceleration changes both the energy and the differential flux because both the energy and the size of the energy interval change. Figure 12 shows the helium spectrum with the deceleration removed, but not the modulation. It is seen that the shape changes relatively little and is still easily fitted by the modulation functions using a slightly different value of A and $R_o$. In the low energy region the deceleration becomes large, but the spread in the energy interval compensates for this effect somewhat leaving the shape changed little.

It does not appear that this effect could explain the increase in the differential helium flux at low energies in itself because although the deceleration effect increases in strength at low energies the suppression due to diffusion does also, generally leading to the type of result shown in Figure 12. Further the proton component does not show a similar effect as would be expected. Hence, apparently the helium nuclei flux increase at very low energies is a function of the difference between the unmodulated proton and helium spectrum. Assuming that K is proportional to BR does not change the argument appreciably since for $r_o V/K$ values greater than 3 the degree of deceleration increases only slowly.

It is also worth mentioning that it is unlikely that the increase in the helium particle flux below 30 MeV/nucleon observed in 1964 and 1965 is due to there being few scattering kinks or cell sizes below the corresponding rigidity for two reasons. Firstly, a proton of the same rigidity has a kinetic energy of about 115 MeV and there is no indication of a break in the spectra at that point. Secondly, a 30 MeV/nucleon helium nucleus has a radius of curvation $3.2 \times 10^5$ km in a 5 gamma field and the satellite magnetic field data analyzed in the manner described earlier extend down to periods as short as 100 sec corresponding to $0.4 \times 10^5$ km. Also, since these low energy particles only appear near solar minimum and the proton to helium ratio is quite small in the energy/nucleon interval where they appear, they are probably not solar. Therefore, although the situation is not yet entirely clear, they are probably in the primary flux.

In looking for other experimental evidence for deceleration or the lack of it, Fichtel and Reames (1966) summarize the several reasons for believing there was not any appreciable Fermi type deceleration, but indicated that for the expansion deceleration case, the experimental data could only argue against large (>25%) decelerations, for medium energy (~150 MeV/nuc.) nuclei and electrons. The two major considerations were the light to medium nuclei ratio hump at 150 MeV/nucleon and the presence of low energy (1 to 10 MeV) electrons. From the analysis here, only a 10 to 20% deceleration is expected for 130 MeV/nucleon particles, and, since B is essentially one for 1 to 10 MeV electrons and the rigidity is sufficiently small so that electrons may be scattered very little and simply follow the field lines. Since $r_o V/K$ is small because K is large; there is also little deceleration for these particles.

## VII. Summary

The solar wind theory of the modulation of galactic cosmic rays now seems to be on reasonably firm ground experimentally and theoretically, being consistent with the recent plasma and field measurements as well as the particle measurements; whereas other well known modulation models now have serious difficulties. It was shown that the solar wind theory parameters determined experimentally from the charged particle data are consistent with the values expected from other knowledge. It was further seen that the available experimental data allow only limited conclusions to be drawn without making any restrictions at all on the particle spectra outside the solar system.

Several different assumptions were then made about the source spectra. The historically popular assumption of similar energy/nucleon spectra at the source for protons and helium nuclei leads to unmodulated cosmic ray spectra which have very high intensities at low energies. In fact, the flux is so large that the cosmic ray energy density would clearly exceed the magnetic field energy density in the galaxy. Further, this result is inconsistent with the helium nuclei energy spectrum derived from the assumption that helium and heavier nuclei with the same charge to mass ratio have the same spectrum at the source. A similar analysis based on similar rigidity spectra at the source for protons and helium nuclei led to a source spectral shape which was more reasonable. The general conclusions were not affected by the type of scattering center distribution assumed over a wide range of choices. Further, the inclusion of secondary protons produced in intergalactic space tended to have a relatively small effect.

The next approach was to assume that a helium spectrum of the type deduced from the helium and heavy nuclei data, assuming similar source spectra, was correct. Given the unmodulated spectrum, the degree of modulation could be calculated for each year, and from these results it was possible to calculate an unmodulated source spectrum which differed from the helium one and led to a proton to helium nuclei ratio which increases with energy/nucleon in the range from 0.1 to 20 BeV/nucleon, if the high energy experimental data are correct.

Finally, the importance of deceleration was considered, and, although the degree of deceleration is somewhat difficult to estimate it was seen that the particular shape of the measured spectrum made the previous analysis insensitive to the deceleration effect.

Thus, although it is not yet possible to have a clearly definitive answer to the problem of modulation, there are a number of important conclusions which can be drawn. Further, it is beginning to appear that the concept of similar source spectra for protons and helium nuclei will have to be abandoned ultimately. On the experimental side, precise proton, helium nuclei and electron spectra as a function of time and distance from the sun over a wide range of energies will provide the basis for a much more exact understanding of the modulation process.

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Figure Captions

Figure 1. Energy distribution of secondary singly charged particles formed in interactions of cosmic rays with nuclei in nuclear emulsions. These secondaries had angles of less than $30^{\circ}$ with respect to a vector pointing downward into the earth and represent data collected during the balloon flights in 1961, 1963, and 1964.

Figure 2. The differential energy/nucleon fluxes for protons measured by various investigators in 1959, 1961, 1963 and 1965.

1959- : Webber and McDonald (1964); 1961- : Present work; 1963- : Present work; 1963- : McDonald and Ludwig (1964); 1963- : Balasubrahmanyan and McDonald (1964); 1963- : Freier and Waddington (1965b); 1963- : Ormes and Webber (1965); 1965- : Balasubrahmanyan et al. (1966); 1965- : Ormes and Webber (1965).

Figure 3. The differential energy/nucleon fluxes for helium nuclei measured by various investigators in 1957, 1961, 1963, and 1965.

1959- : Webber and McDonald (1964); 1961- Present Work; 1963- Present work; 1963- : Balasubrahmanyan and McDonald (1964); 1963- : Fan et al. (1965);

1963-: Freier and Waddington (1965b); 1963-/ : Ormes and Webber (1965); 1965-: Balasubrahmanyan et al. (1966); 1965-: Comstock et al. (1966); 1965-: Ormes and Webber (1965).

Figure 4. Proton to helium nuclei ratio determined from the 1963 measurements shown in Figures 2 and 3. The curves indicate the theoretical ratio for the parameters given in the figure assuming similar energy/nucleon spectra for protons and helium nuclei at the source.

Figure 5. Unmodulated spectra and source spectra obtained using the 1963 observed spectra and assuming 2.8 g/cm$^2$ of interstellar matter, similar velocity spectra at the source for protons and helium nuclei, and the indicated parameters.

Figure 6. Proton to helium nuclei ratios observed in 1963 as obtained from Figures 2 and 3, plotted as a function of proton rigidity. The curves show the theoretical values which were obtained for the indicated parameters. Note: the helium flux value which is compared to the proton value is measured at a different rigidity; see the text for an explanation.

Figure 7. Source spectra and modulated spectra obtained using the observed 1963 spectra and assuming 2.5 g/cm$^2$ of interstellar matter, similar rigidity spectra at the source for

protons and helium nuclei, and the parameters indicated in the figure.

1961-△ alpha, ▲ protons: present work; 1962-○ alpha, ● protons: present work; 1963 ---- averaged 1963 data, see Figures 1 and 2 for references; 1965-▽ alpha: Comstock et al., (1965); 1965-◇ alphas, ◆ protons: Balasubrahmanyan et al., (1966); □ alpha, ■ protons: Ormes and Webber (1965).

Figure 8a. Unmodulated differential spectra for protons and helium nuclei with the secondary singly charged particle correction (solid line) and without it (dashed line) assuming a source spectral shape of the form $C/W^{2.5}$, where W is the total energy per nucleon, an interstellar path length of 2.8 g/cm$^2$, and proton to helium nuclei ratios at the source in this energy region of three (B) and five (A).

Figure 8b. Unmodulated differential spectra for protons and helium nuclei with the secondary singly charged particle correction (solid line) and without it (dashed line) assuming a source spectral shape of the form deduced earlier assuming similar source rigidity spectra (See Figure 7), an interstellar path length of 2.8 g/cm$^2$, and a proton to helium nuclei ratio of five.

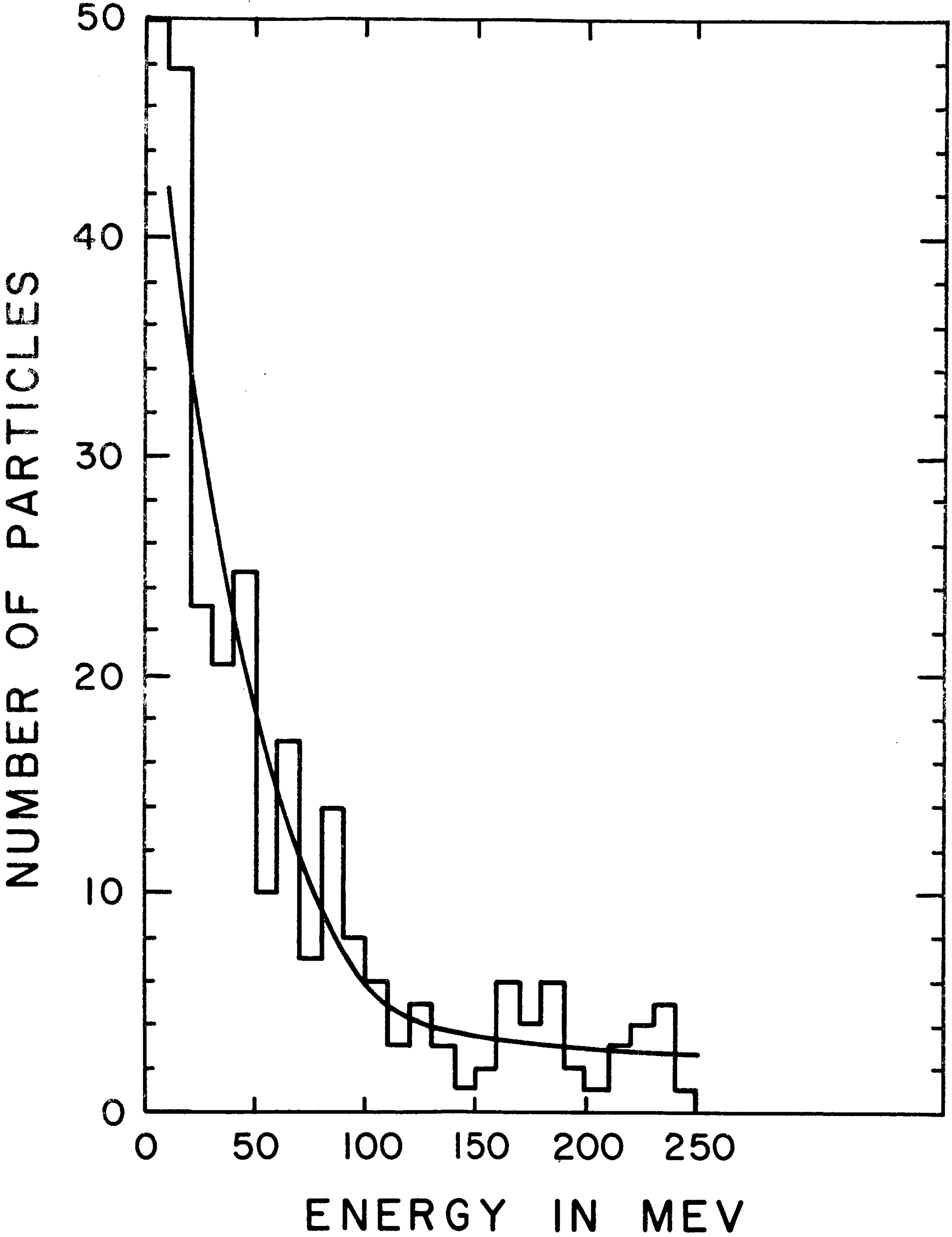
Figure 9a. Calculated helium spectra obtained for 1965, 1963, 1961, and 1959 assuming a source spectral shape of the form $C/W^{2.5}$, an interstellar path of 2.8 $g/cm^2$, a modulation function of the form G of table III, and the parameters given in the figure.

Figure 9b. Calculated helium spectra obtained for 1965, 1963, 1961, and 1959 assuming a source spectral shape of the form $C/W^{2.7}$, an interstellar path of 2.8 $g/cm^2$, a modulation function of the form G of table III, and the parameters given in the figure.

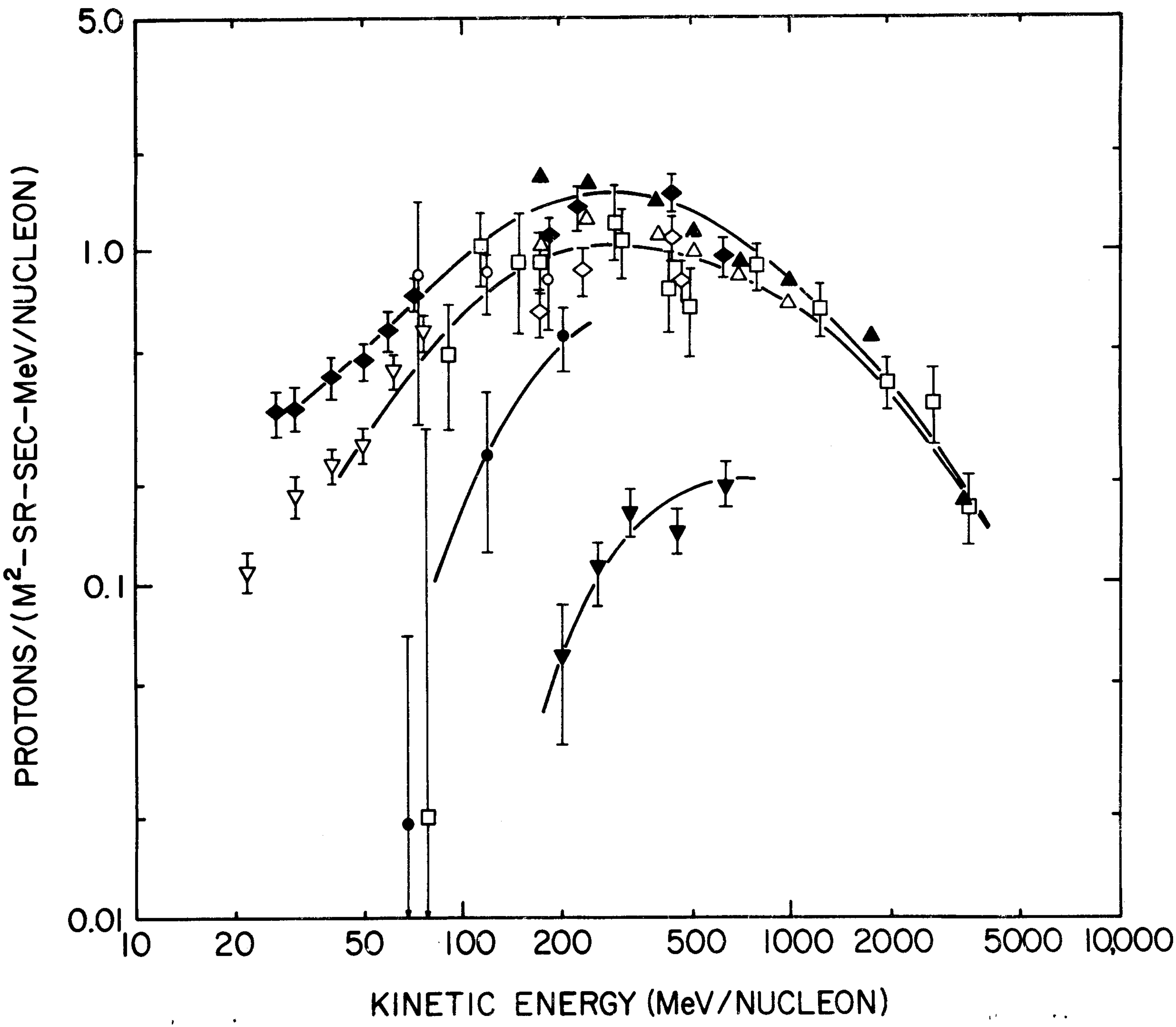
Figure 10. Unmodulated proton spectrum deduced from an analysis of the helium nuclei data (see text) and the calculated modulated proton spectra for 1965, 1963, 1961, and 1959 using the parameters shown which are the same as those deduced for the helium nuclei for $\alpha$=2.5.

Figure 11. Derived source spectrum for helium nuclei (B) and protons (D) using the unmodulated proton spectrum (C) of figure 10 and the unmodulated helium spectrum (A) discussed in section V C.

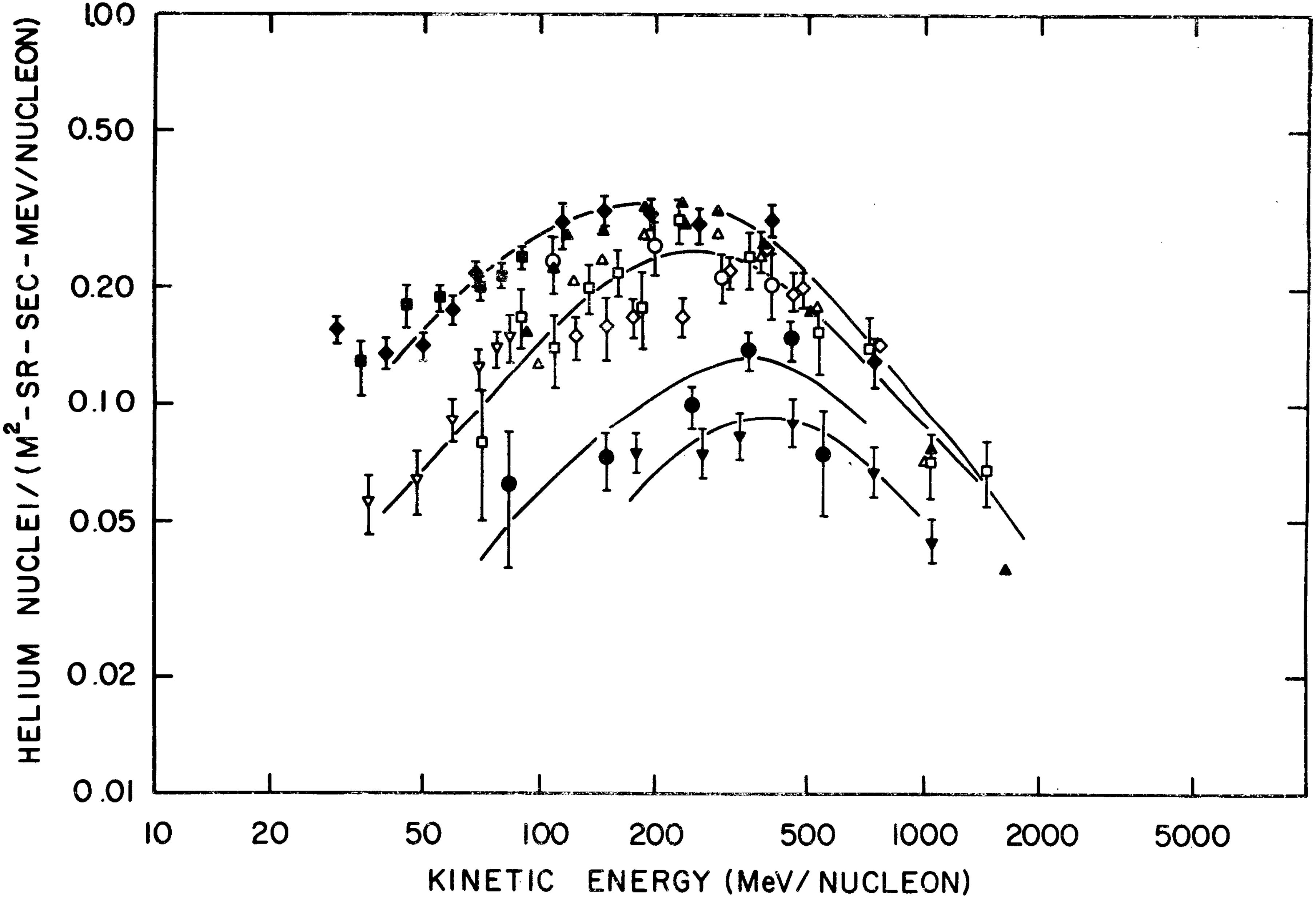
Figure 12 Smooth fit to observed 1963 modulated helium nuclei spectrum (solid line) and curve (dashed line) showing an estimate of what the spectrum would have been if there had been only convection-diffusion modulation and no deceleration (see text).



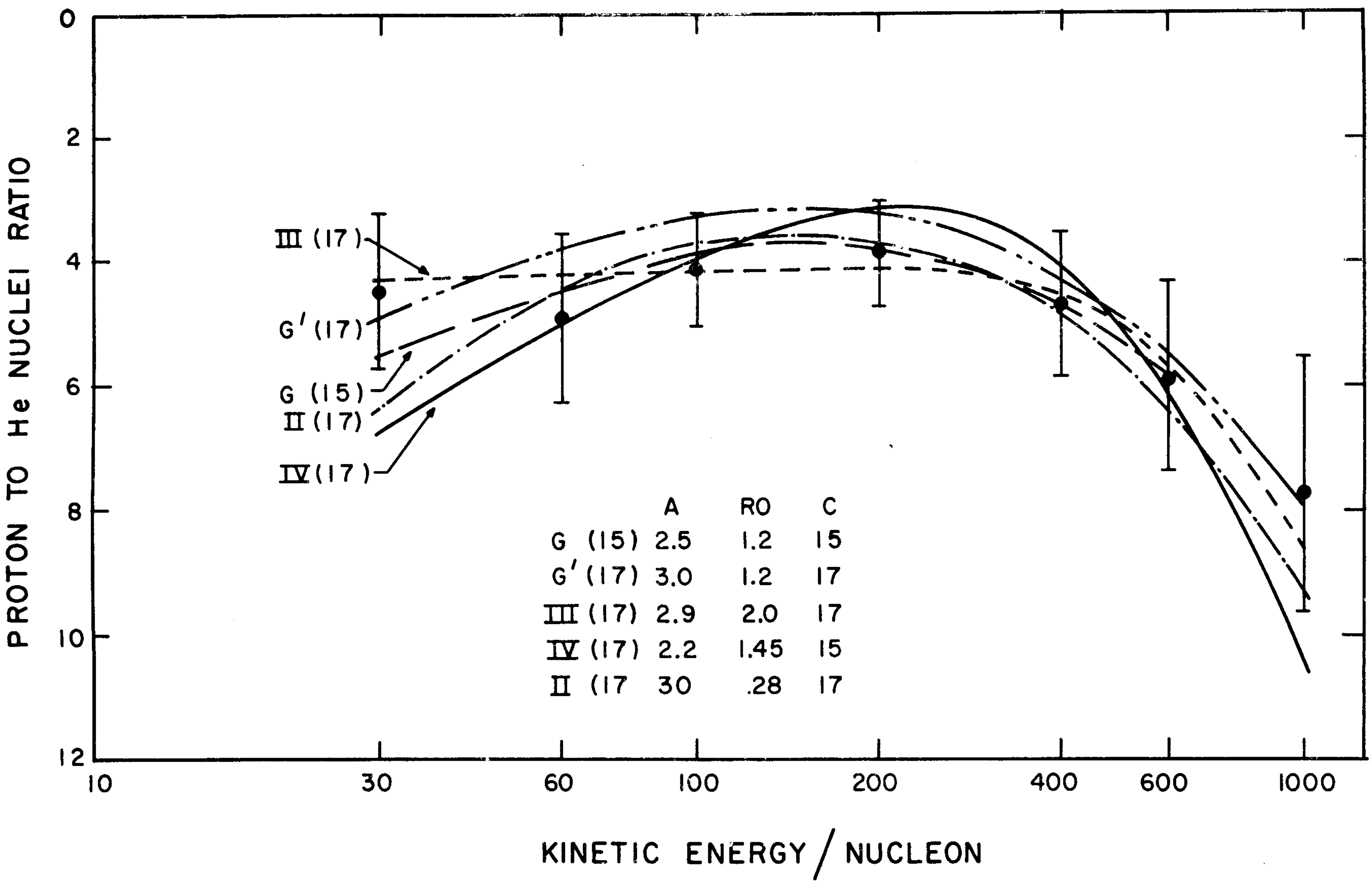
NUMBER OF PARTICLES
ENERGY IN MEV
0
10
20
30
40
50
0
50
100
150
200
250



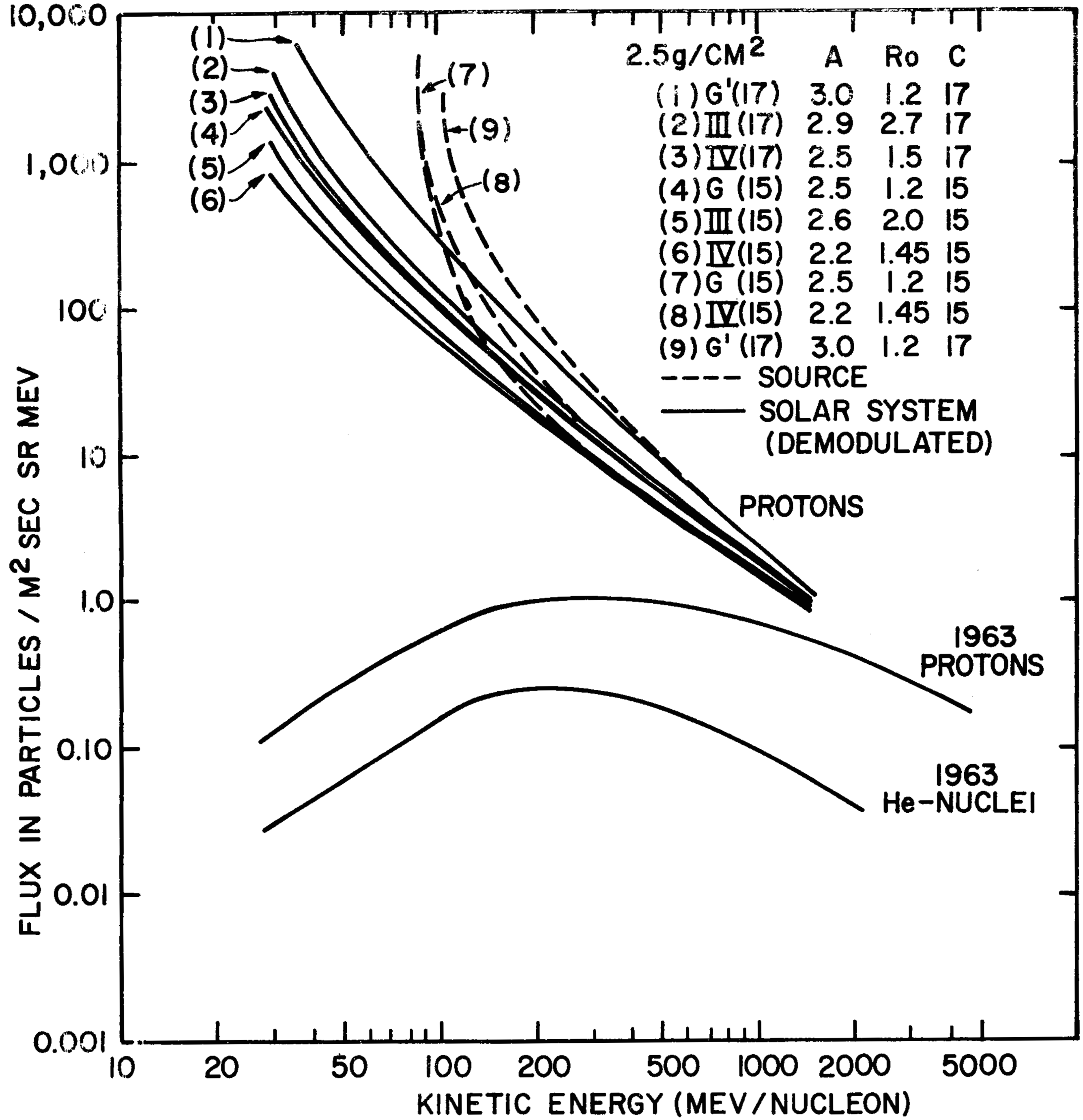
5.0
1.0
0.1
0.01
10
20
50
100
200
500
1000
2000
5000
10,000
PROTONS/(M²-SR-SEC-MeV/NUCLEON)
KINETIC ENERGY (MeV/NUCLEON)



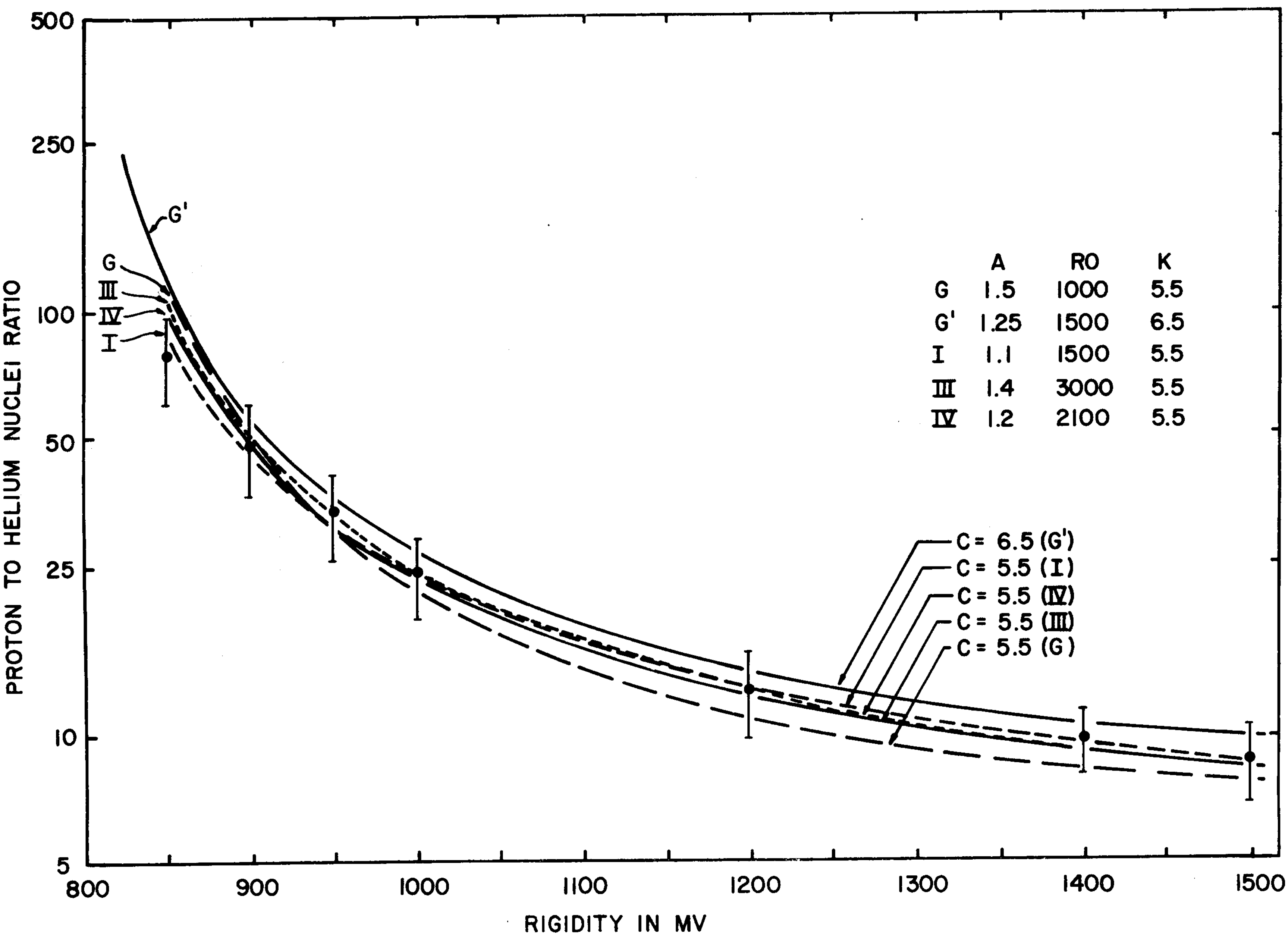
HELIUM NUCLEI/(M²-SR-SEC-MEV/NUCLEON)
1.00
0.50
0.20
0.10
0.05
0.02
0.01
10
20
50
100
200
500
1000
2000
5000
KINETIC ENERGY (MeV/NUCLEON)

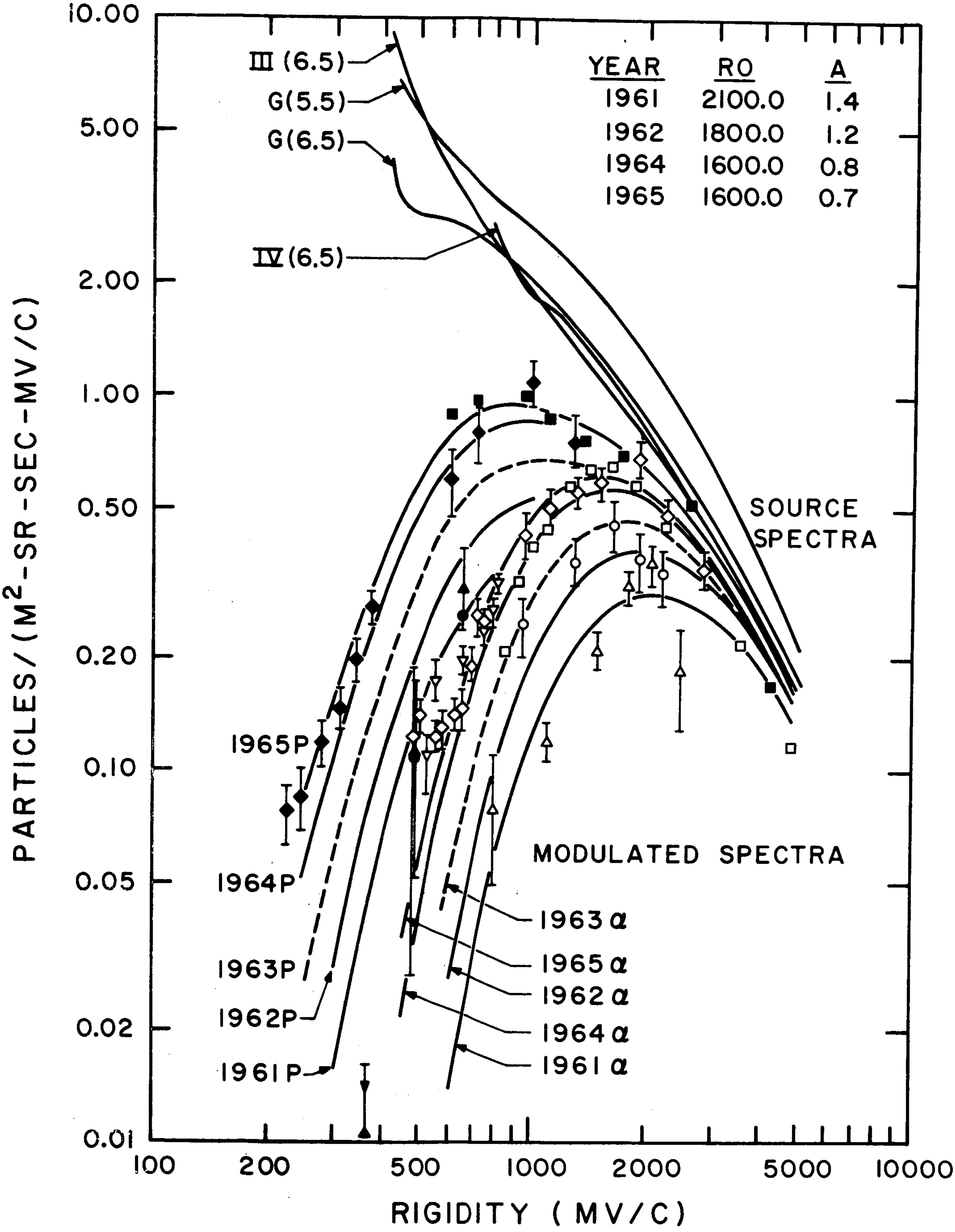


PROTON TO He NUCLEI RATIO
0
2
4
6
8
10
12
10
30
60
100
200
400
600
1000
KINETIC ENERGY / NUCLEON
III (17)
G′ (17)
G (15)
II (17)
IV (17)
A RO C
G (15) 2.5 1.2 15
G′ (17) 3.0 1.2 17
III (17) 2.9 2.0 17
IV (17) 2.2 1.45 15
II (17 30 .28 17

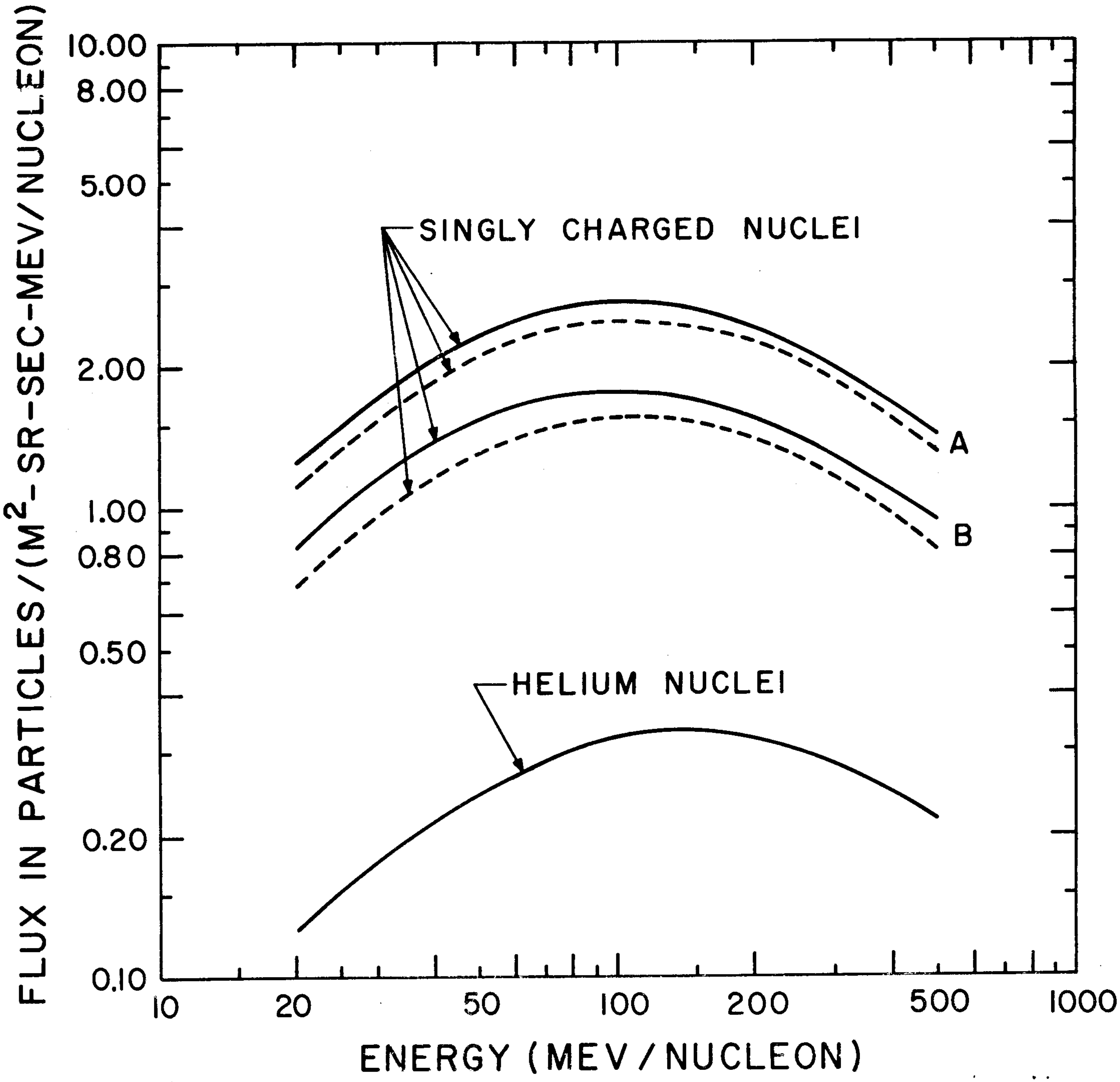


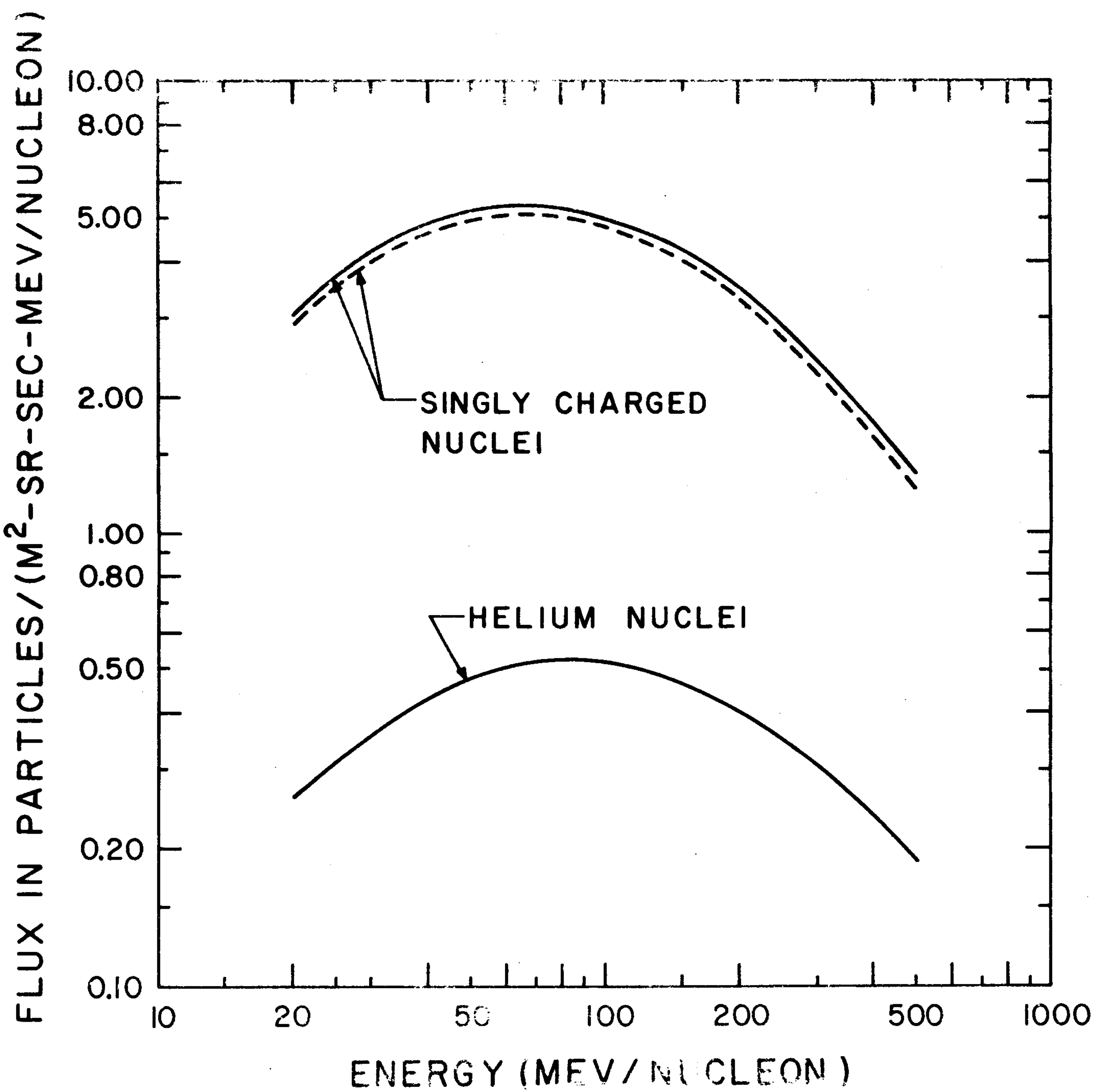
2.5g/CM²
A
Ro
C
(1) G'(17) 3.0 1.2 17
(2) III (17) 2.9 2.7 17
(3) IV (17) 2.5 1.5 17
(4) G (15) 2.5 1.2 15
(5) III (15) 2.6 2.0 15
(6) IV (15) 2.2 1.45 15
(7) G (15) 2.5 1.2 15
(8) IV (15) 2.2 1.45 15
(9) G' (17) 3.0 1.2 17
SOURCE
SOLAR SYSTEM (DEMODULATED)
PROTONS
1963 PROTONS
1963 He-NUCLEI
(1)
(2)
(3)
(4)
(5)
(6)
(7)
(9)
(8)
10,000
1,000
100
10
1.0
0.10
0.01
0.001
10
20
50
100
200
500
1000
2000
5000
FLUX IN PARTICLES / M² SEC SR MEV
KINETIC ENERGY (MEV/NUCLEON)

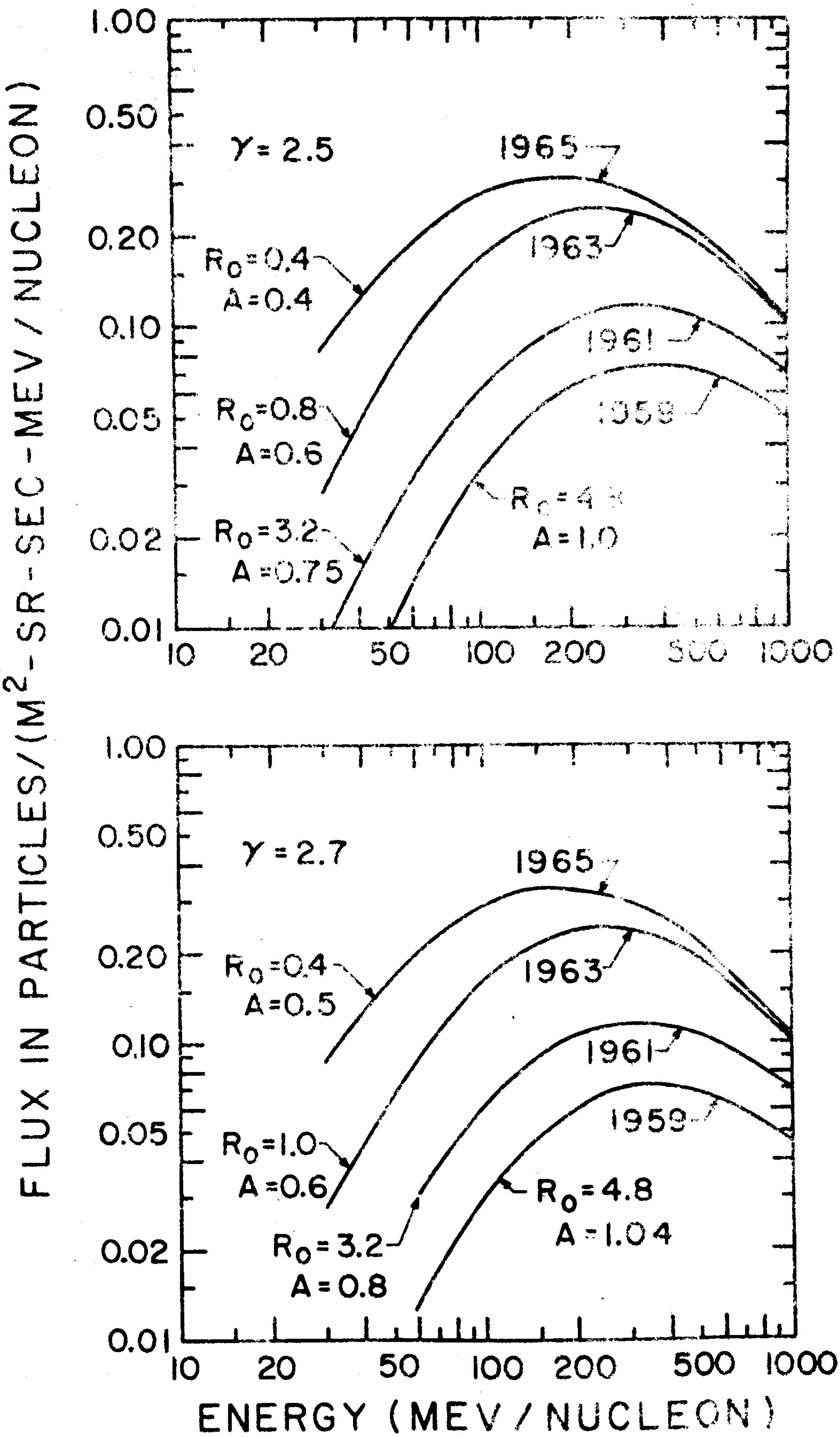

PROTON TO HELIUM NUCLEI RATIO
500
250
100
50
25
10
5
800
900
1000
1100
1200
1300
1400
1500
RIGIDITY IN MV
G'
G
III
IV
I
A RO K
G 1.5 1000 5.5
G' 1.25 1500 6.5
I 1.1 1500 5.5
III 1.4 3000 5.5
IV 1.2 2100 5.5
C = 6.5 (G')
C = 5.5 (I)
C = 5.5 (IV)
C = 5.5 (III)
C = 5.5 (G)

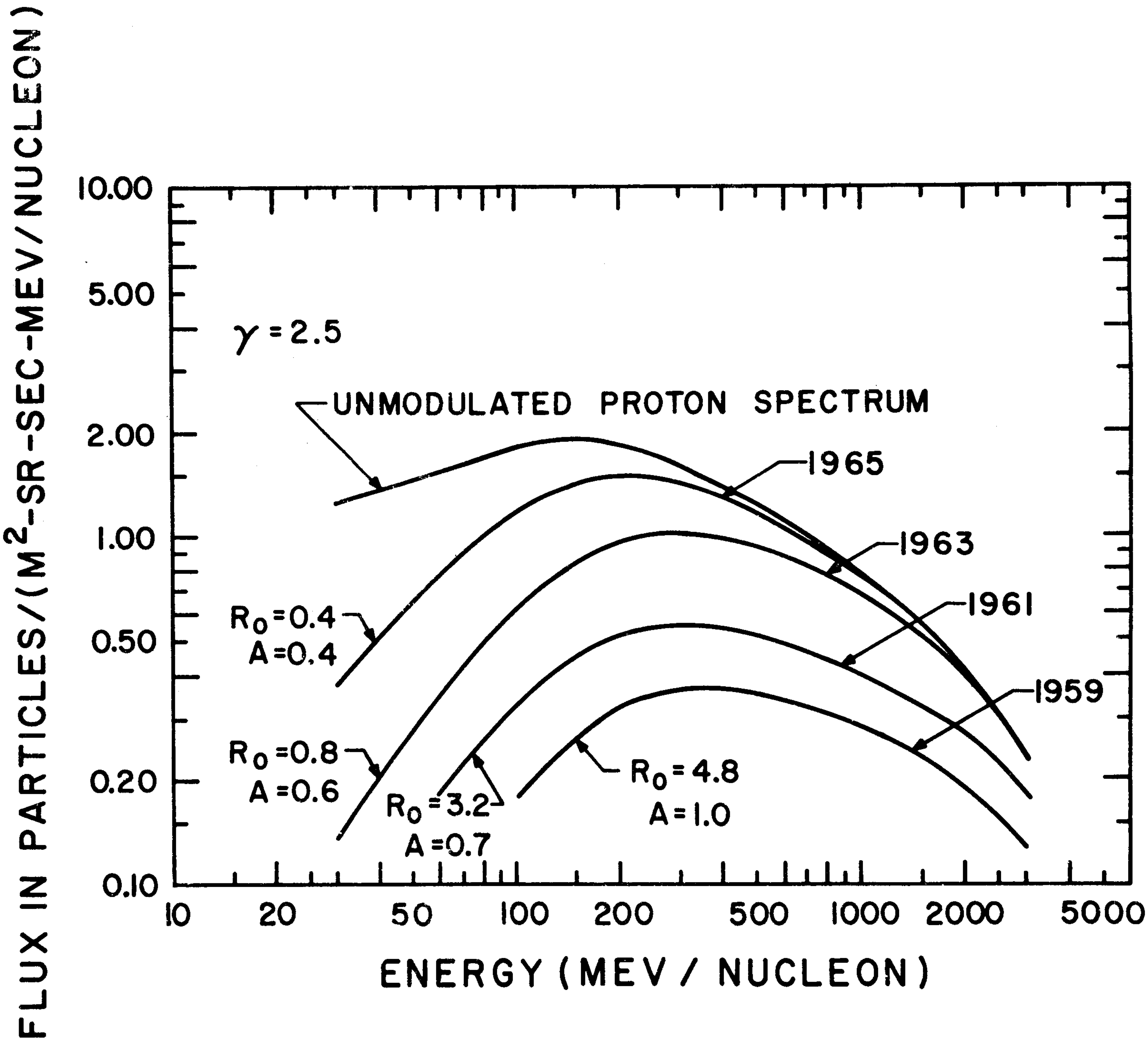


YEAR RO A
1961 2100.0 1.4
1962 1800.0 1.2
1964 1600.0 0.8
1965 1600.0 0.7
III (6.5)
G(5.5)
G(6.5)
IV (6.5)
SOURCE SPECTRA
MODULATED SPECTRA
1965P
1964P
1963P
1962P
1961P
1963 α
1965 α
1962 α
1964 α
1961 α
PARTICLES/(M²-SR-SEC-MV/C)
RIGIDITY (MV/C)
10.00
5.00
2.00
1.00
0.50
0.20
0.10
0.05
0.02
0.01
100
200
500
1000
2000
5000
10000

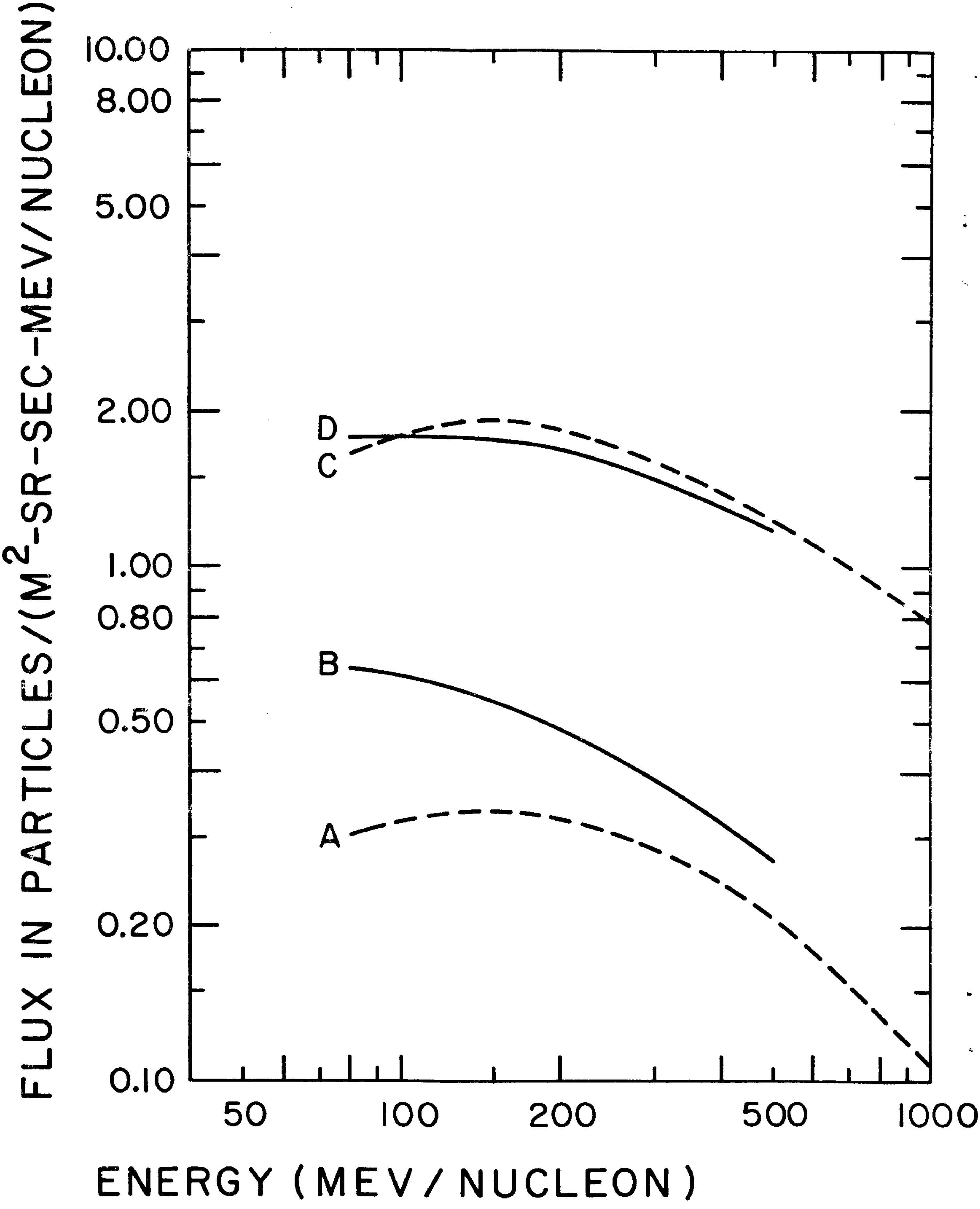

FLUX IN PARTICLES / (M2- SR-SEC-MEV/ NUCLEON)
10.00
8.00
5.00
2.00
1.00
0.80
0.50
0.20
0.10
SINGLY CHARGED NUCLEI
A
B
HELIUM NUCLEI
10
20
50
100
200
500
1000
ENERGY (MEV / NUCLEON)

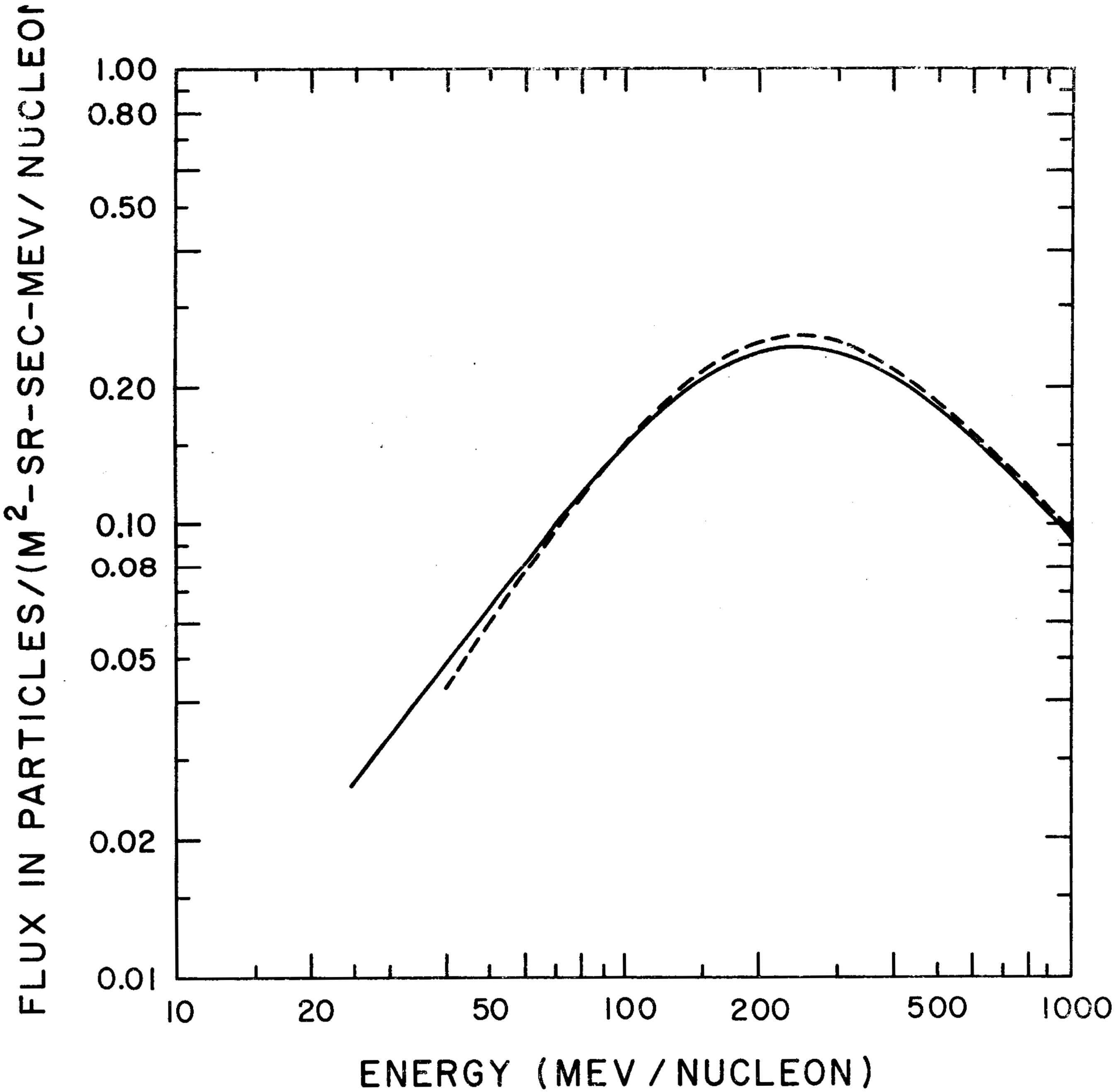

FLUX IN PARTICLES/(M2-SR-SEC-MEV/NUCLEON)
10.00
8.00
5.00
2.00
1.00
0.80
0.50
0.20
0.10
SINGLY CHARGED
NUCLEI
HELIUM NUCLEI
10
20
50
100
200
500
1000
ENERGY (MEV/NUCLEON)


FLUX IN PARTICLES/(M$^2$-SR-SEC-MEV/NUCLEON)
1.00
0.50
0.20
0.10
0.05
0.02
0.01
$\gamma$ = 2.5
1965
1963
1961
1959
$R_0$=0.4
A=0.4
$R_0$=0.8
A=0.6
$R_0$=3.2
A=0.75
A=1.0
10
20
50
100
200
500
1000
$\gamma$ = 2.7
1965
1963
1961
1959
$R_0$=0.4
A=0.5
$R_0$=1.0
A=0.6
$R_0$=3.2
A=0.8
$R_0$=4.8
A=1.04
ENERGY (MEV/NUCLEON)



$\gamma = 2.5$
UNMODULATED PROTON SPECTRUM
1965
1963
1961
1959
$R_0 = 0.4$
$A = 0.4$
$R_0 = 0.8$
$A = 0.6$
$R_0 = 3.2$
$A = 0.7$
$R_0 = 4.8$
$A = 1.0$
10.00
5.00
2.00
1.00
0.50
0.20
0.10
10
20
50
100
200
500
1000
2000
5000
FLUX IN PARTICLES/($M^2$-SR-SEC-MEV/NUCLEON)
ENERGY (MEV / NUCLEON)


FLUX IN PARTICLES/($M^2$-SR-SEC-MEV/NUCLEON)
10.00
8.00
5.00
2.00
1.00
0.80
0.50
0.20
0.10
D
C
B
A
50
100
200
500
1000
ENERGY (MEV/NUCLEON)


FLUX IN PARTICLES/(M2-SR-SEC-MEV/ NUCLEON
1.00
0.80
0.50
0.20
0.10
0.08
0.05
0.02
0.01
10
20
50
100
200
500
1000
ENERGY (MEV / NUCLEON)